Enrollment No. ____________

Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous), Rajkot
Affiliated to Saurashtra University, Rajkot

SEMESTER END EXAMINATION APRIL - 2018

M.Sc. Mathematics

16PMTCC20 - DIFFERENTIAL GEOMETRY

Duration of Exam – 3 hrs ***Semester – IV*** ***Max. Marks – 70***

***Part A* (5X2= 10 marks)**
Answer **ALL** questions

1. Define with example: Function of class C^k.
2. Define with example: Regular curve.
3. Define tangent vector field.
4. Define curvature of a curve.
5. Define a simple surface.

***Part B* (5X5= 25 marks)**
Answer **ALL the** questions

6a. Is the curve $(t^4, t^2, 2t+5)$ regular? Justify your answer.

OR

6b. Define right circular helix and find the arc length of the same.

7a. Define reparametrization of a curve. If $g:[c,d]\to[a,b]$ is a reparametrization of a curve segment $\sim:[a,b]\to R^3$ then prove that the length of $\sim$ is equal to the length of $\;=\sim\circ g$.

OR

7b. Is the curve $\sim(t)=\left(\cos t, \cos^2 t, \sin t\right)$ regular? If so then find the equation of tangent line at $t=\frac{®}{4}$.

8a. Show that the length of the curve $\sim(t)=\left(2a\left(\sin^{-1}t+t\sqrt{1-t^2}\right), 2at^2, 4at\right)$ between the points $t=t_1$ to $t=t_2$ is $4a\sqrt{2}\left(t_2-t_1\right)$.

OR

8b. Show that $\sim(s)=\left(\frac{(1+s)^{\frac{3}{2}}}{3}, \frac{(1-s)^{\frac{3}{2}}}{3}, \frac{s}{\sqrt{2}}\right)$ is a unit speed curve and compute its Frenet – Serret appartus.

9a. Show that $\sim(s)=\frac{1}{2}\left(\cos^{-1}s, s\sqrt{1-s^2}, 1-s^2, 0\right)$ is a unit speed curve and compute its Frenet – Serret appartus.

OR

9b. Find the arc length of the curve $\sim(t)=\left(a\cos t, a\sin t, at\tan\sim\right)$.

10a Identify the curve $x^2+y^2-8x-4y-16=0$ and find the curvature of the same.

OR

10b State and prove Frenet-Serret theorem

***Part C* (5X7= 35 marks)**

Answer **ALL** the questions

11a. If $x:u\to R^3$ is a simple surface and $f:v\to u$ is a co-ordinate transformation such that $y=x\circ f$ then prove that

i) The tangent plane to the simple surface *x* at *P* = *x*(*f*(*a*,*b*)) is equal to the tangent plane to the simple surface *y* at *P* = *y*(*a*,*b*).

ii) The normal to the surface *x* at *P* is same as the normal to the surface *y* at *P* except possibly it may have the opposite sign.

OR

11b. Prove that: The set of all tangent vectors to a simple surface $x:u\to R^3$ at *P* is a vector space.

12a. Let $\sim(s)$ be a unit speed curve whose image lies on a sphere of radius *r* and centre *m* then show that $k\neq 0$. Also if $² \neq 0$ then $\sim - m = -° N - °'±$ and $r^2 = °^2 + (°'±)^2$ (where $° = \frac{1}{k}$ and $± = \frac{1}{²}$).

OR

12b. Let $f:X\to R^3$ be a simple surface and $f:v\to u$ is a co-ordinate transformation then prove that $y=X\circ f:v\to R^3$ is also a simple surface.

13a. Let $x:u\to R^3$ be a simple surface then prove that

i) $x_{ij}=L_{ij}n+\sum_k \Gamma_{ij}^k x_k$

ii) For any unit speed curve $¥(S)=x(¥'(S),¥^2(S))$, $k_n=\sum_{i,j} L_{ij}(¥^i)'(¥^i)'$ and

$$k_g S=\sum_k\left[(¥^k)''+\sum_{i,j}\Gamma_{ij}^k(¥^i)'(¥^j)'\right]x_k .$$

OR

13b. Prove in the usual notations: $d_{ij}^l=\frac{1}{2}\sum_{k=1}^{2} g^{kl}\left(\frac{\partial g_{ik}}{\partial u^j}+\frac{\partial g_{kj}}{\partial u^i}-\frac{\partial g_{ij}}{\partial u^i}\right)$

14a. Define tangent space and normal space. Prove in the usual notations the relation $k^2 = k_n^2 + k_g^2$

OR

14b. Show that a unit speed curve is a helix iff there is a constant *c* such that $² = c©$.

15a. Define Monge patch and compute the first fundamental forms for the same. Also obtain the matrix (g_{ij}).

OR

15b. Define Monge patch and compute the coefficients of second fundamental form and Christoffel symbols for the same.
